Strong-field QED with tightly focused femtosecond pulses

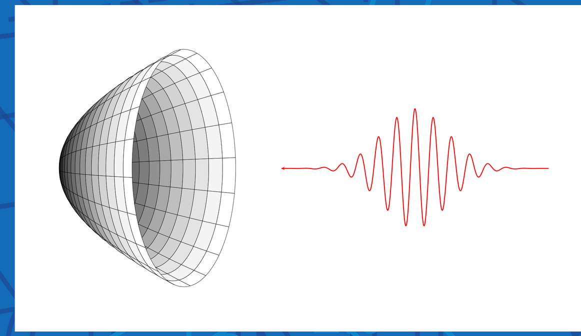
Application to vacuum four-wave mixing

Joey Dumont*, François Fillion-Gourdeau, Catherine Lefebvre, Denis Gagnon and Steve MacLean

Université du Québec – INRS-ÉMT, Varennes, Québec, Canada

* joey.dumont@emt.inrs.ca

INRS UNIVERSITÉ DE RECHERCHE



High-intensity lasers

Recent advances in high-power laser systems have made it possible to obtain intensities on the order of $10^{22}\,\mathrm{W/cm^2}$, opening up the possibility of an experimental detection of strong-field quantum electrodynamics (SF-QED) effects. These high intensities can be achieved by [1]

- compressing a laser pulse to **femtosecond** durations;
- **strongly focusing** the beam (λ^3 **focal volume**).

Experimentally, tightly focused fields are generated with **high numerical aperture** parabolic mirrors. The fields reflected from this type of mirror acquire a **complex spatial structure**. This affects the **design** of SF-QED experiments, as the observables are impacted in a non-trivial way [2].

Strong-field QED processes

Goal 1: Develop a realistic model of the reflection process of femtosecond pulses off high numerical aperture mirrors.

Goal 2: Characterize the observables of SF-QED with the model.

Stratton-Chu diffraction integrals

A realistic model for tightly focused fields must be developed because:

- paraxial models **fail** in this regime;
- the **shape** of the mirror has a distinct effect on the focused fields;

The **Stratton-Chu** equations

- are **exact solutions** of Maxwell's equations;
- take into account the mirror geometry;

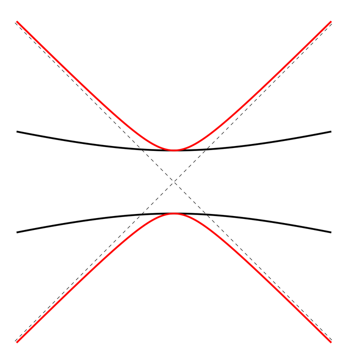


Fig. 1: Tightly focused fields have a large divergence angle.

Four-wave mixing in vacuum

We illustrate our method by modeling the effect of four-wave mixing in vacuum using the **Euler-Heisenberg** effective theory. Vacuum fluctuations are interpreted as a **non-linear polarization**.

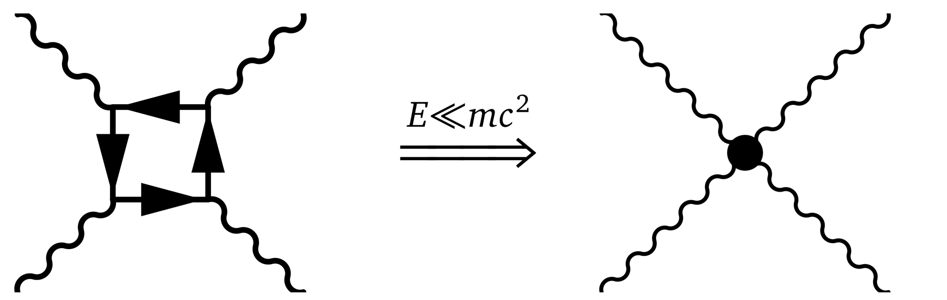


Fig. 2: At low energies, $E \ll mc^2$, and for slowly varying fields, wave mixing in vacuum can be described by the Euler-Heisenberg effective theory. At first-order in the field strength, they yield quantum corrections to Maxwell's equations in the form of polarization and magnetization terms.

Four-wave mixing experiment

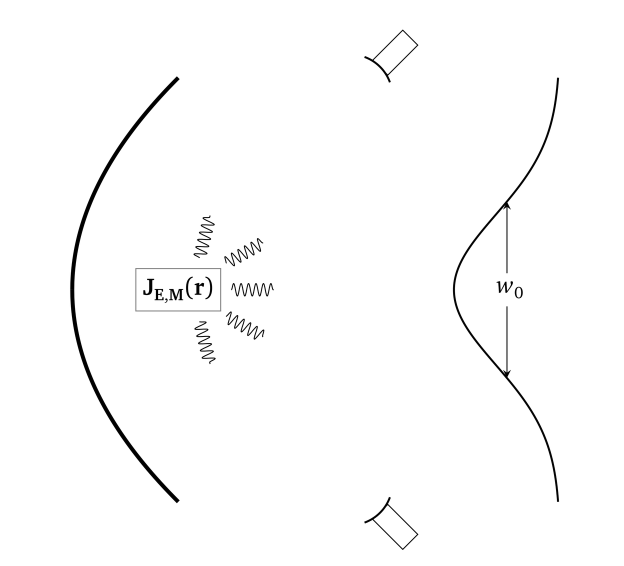


Fig. 3: Pictorial schematic of a SF-QED experiment.

The theoretical simulation of a wave mixing experiment proceeds as follows:

1. Characterization of the incident laser beam;
2. Computation of the **strongly focused field** in the focal spot using the Stratton-Chu formalism;
3. Evaluation of the Euler-Heisenberg **polarization** and **magnetization** in the focal spot;
4. Solution of the **non-linear propagation** problem;
5. Determination of the final **photon energies and angular distribution** on a spherical surface (detectors).

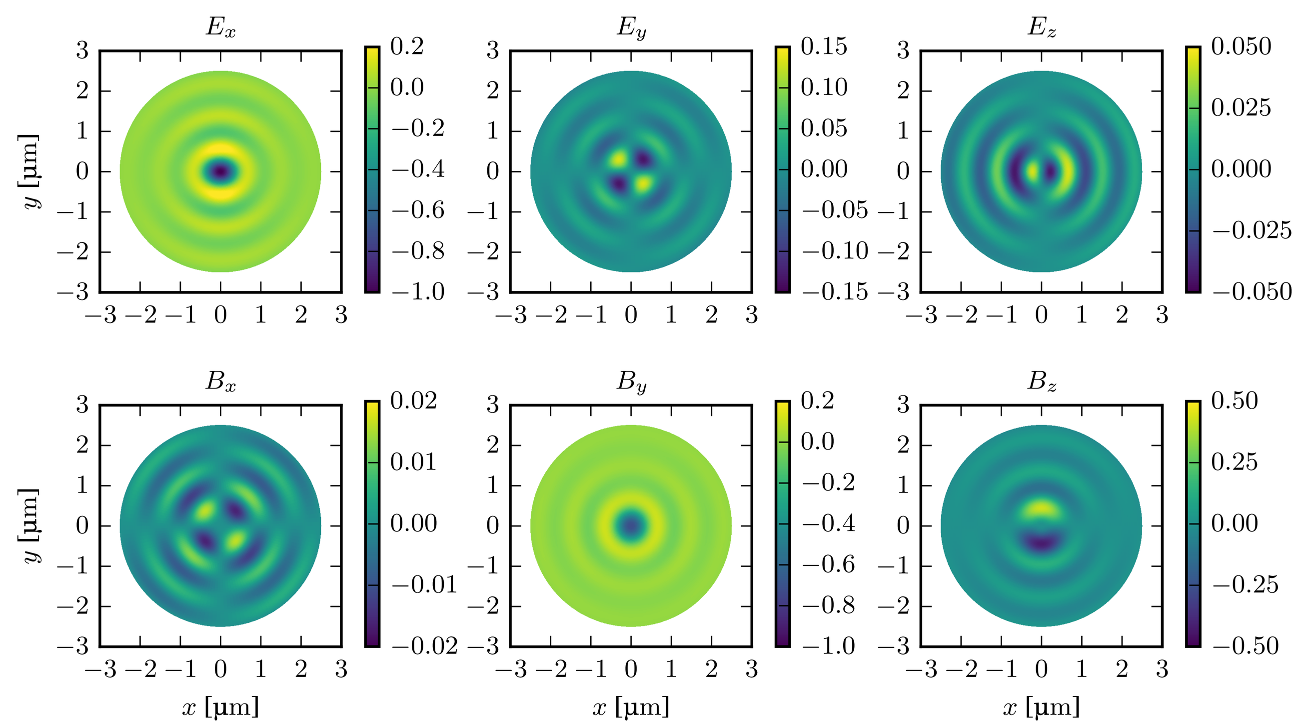


Fig. 5: Components of the electromagnetic field in the focal spot of high numerical aperture mirror (NA = 1) illuminated by a linearly polarized Gaussian beam.

Numerical method

The reflection of the incident laser beam is modeled by the **Stratton-Chu integral equations**. We have developed an **efficiently parallelized** implementation of this framework [4] that can accommodate

- arbitrary mirror geometries;
- incident fields with arbitrary spatial dependence;
- arbitrary spectral power and spectral phase.

It can model the reflection of **temporally short pulses** that inherently possess a broad spectrum. It can also represent the **complicated spatial profile** of a high-power laser.

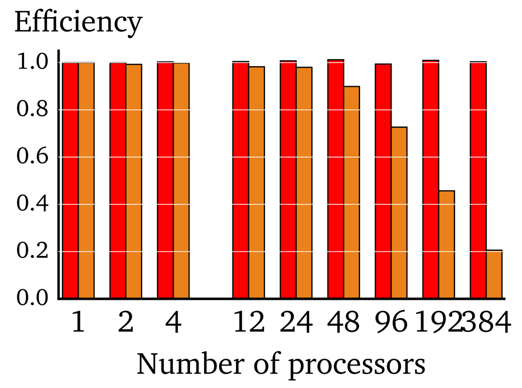


Fig. 4: Parallel efficiency of our implementation of the Stratton-Chu formalism.

For a single Fourier component, the Stratton-Chu equations are

$$\mathbf{E}'_{\mathrm{ref}}(\mathbf{r}',k) = 2\iint_S \{ik(\hat{\mathbf{n}}\times\mathbf{B}_{\mathrm{inc}})g + (\hat{\mathbf{n}}\cdot\mathbf{E}_{\mathrm{inc}})\nabla g\}\,dS + \frac{2}{ik}\oint_{\partial S}\nabla g\,[\hat{\mathbf{n}}\times(\hat{\mathbf{n}}\times\mathbf{B}_{\mathrm{inc}})]\cdot d\boldsymbol{\ell},$$

$$\mathbf{B}'_{\mathrm{ref}}(\mathbf{r}',k) = 2\iint_S (\hat{\mathbf{n}}\times\mathbf{B}_{\mathrm{inc}})\times\nabla g\,dS.$$

The intense laser fields in the focal spot give rise to a non-linear vacuum described by the Euler-Heisenberg theory

$$\mathbf{P}(\mathbf{r},t) = \frac{4\alpha^2}{45E_S^3}\left\{2\left[\mathbf{E}^2-\mathbf{B}^2\right]\mathbf{E}+7\left[\mathbf{E}\cdot\mathbf{B}\right]\mathbf{B}\right\},$$

$$\mathbf{M}(\mathbf{r},t) = \frac{4\alpha^2}{45E_S^3}\left\{-2\left[\mathbf{E}^2-\mathbf{B}^2\right]\mathbf{B}+7\left[\mathbf{E}\cdot\mathbf{B}\right]\mathbf{E}\right\}.$$

Wave mixing results in a high-power laser

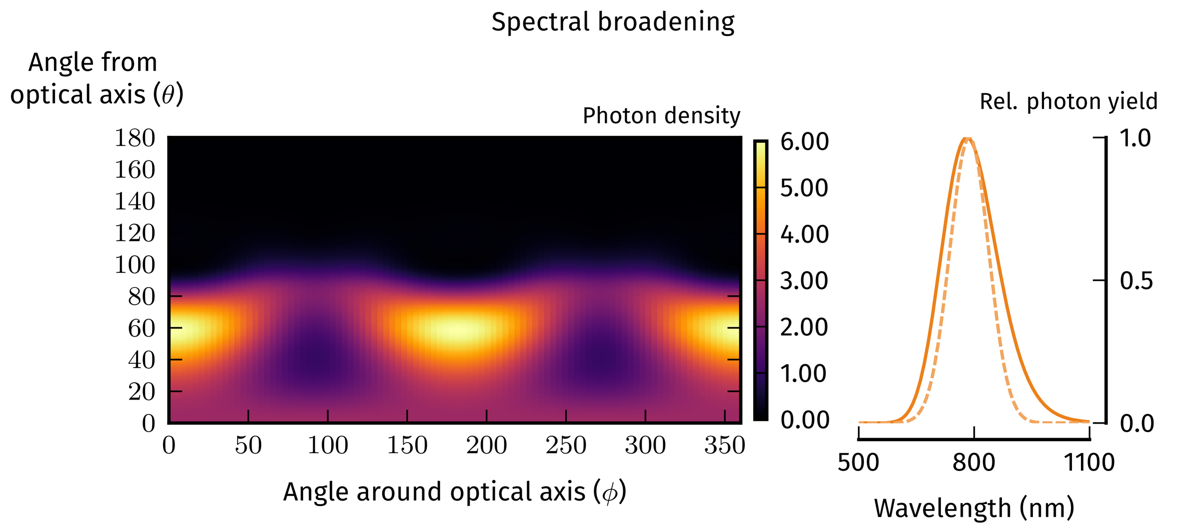


Fig. 5.(a) Photons emitted in the same energy range as the incident beam exhibit a broader spectrum. The emittance peaks follow the divergence of the tightly focused beam, but are broad in the azimuthal direction.

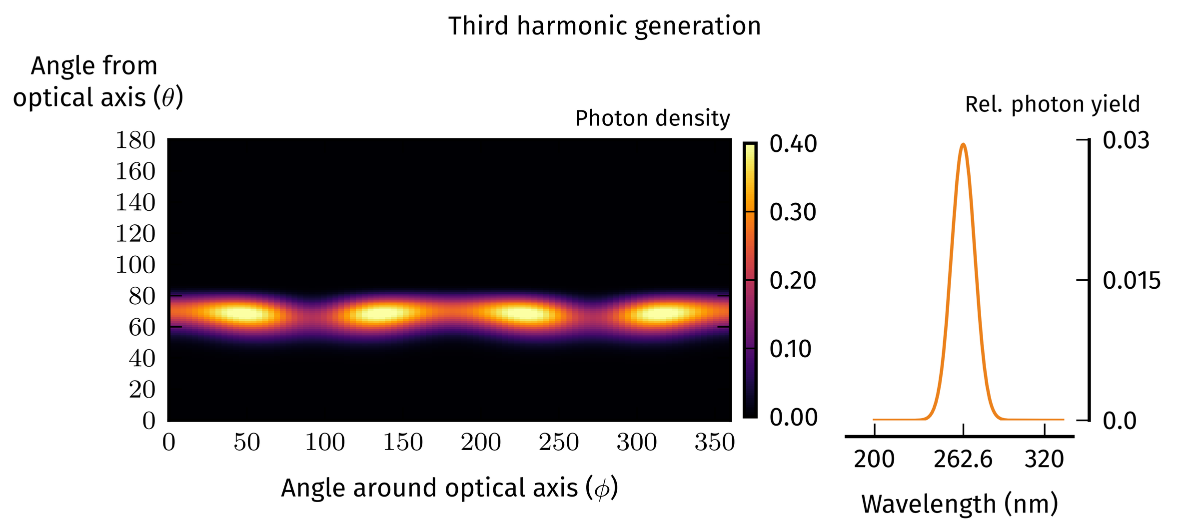


Fig. 5.(b) Third harmonic generation is less efficient than the spectrum broadening, but is emitted in a much tighter cone around the optical axis. The emittance peaks also follow the divergence of the tightly focused beam.

Summary

1. Tightly focused fields such as those generated by **high numerical aperture** mirrors **cannot** be described by the **paraxial approximation**.
2. We have implemented a **scalable, efficient** algorithm based on **exact** solutions of Maxwell's equations that can model the **reflection** of femtosecond laser pulses.
3. This realistic model of tightly focused fields can be used to **guide the design** of SF-QED experiments.
4. We have illustrated this by **simulating a four-wave mixing experiment**. This resulted in a **precise determination** of the energy spectrum and angular distribution of the generated photons.

Outlooks

Optimization

The Stratton-Chu model can used to optimize the four-wave mixing with given experimental limitations. It should be possible to engineer the following parameters to maximize the signal:

- the incident spectral phase;
- the mirror geometry;
- the wavefront of the incident pulse.

Other SF-QED processes

Our realistic model could have an impact in the design of experiments that aim to detect other SF-QED effects [6–9]

- Schwinger pair production;
- Breit-Wheeler pair production;
- Non-linear Compton scattering.

References

[1] G. A. Mourou, T. Tajima, and S. V. Bulanov, Rev. Mod. Phys. **78**, 309 (2006).
[2] F. Karbstein and R. Shaisultanov, Phys. Rev. D **91**, 085027 (2015).
[3] R. Battesti and C. Rizzo, Rep. Prog. Phys. **76**, 016401 (2013).
[4] J. Dumont, F. Fillion-Gourdeau, C. Lefebvre, D. Gagnon and S. MacLean, in preparation (2016).
[5] F. Fillion-Gourdeau, C. Lefebvre, and S. MacLean, Phys. Rev. A **91**, 031801(R) (2015).
[6] J. Dumont *et al*, in preparation (2016).
[7] A. M. Fedotov, Laser Phys. **19**, 214 (2009).
[8] A. Titov *et al*, Phys. Rev. A **87**, 042106 (2013).
[9] F. Mackenroth and A. Di Piazza, Phys. Rev. A **83**, 032106 (2011).

Fonds de recherche Nature et technologies Québec
compute canada | calcul canada