Modelling Tightly Focused Electromagnetic Fields

Stratton-Chu Formulation of Maxwell's Equations

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INRS-ÉMT

June 22-26, 2015

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INRS UNIVERSITÉ DE RECHERCHE

Physical Motivation

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Goal

Observe Quantum Electrodynamics (QED) effects with ultra-intense laser fields.

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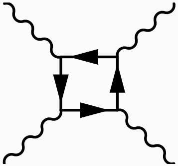
Photon propagation affected by vacuum fluctuations.



Four photons interacting through vacuum flucutations.

G. V. Dunne, “Heisenberg-Euler Effective Lagrangians : Basics and Extensions,” 82 (2004).

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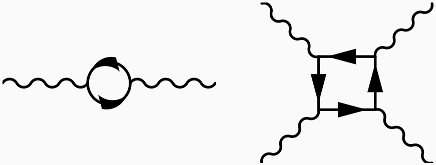
Why lasers?

- Probe photon-photon interactions directly.



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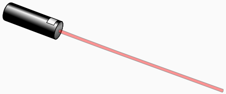
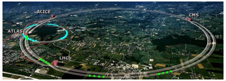
- Way cheaper.

Extreme Light Infrastructure.





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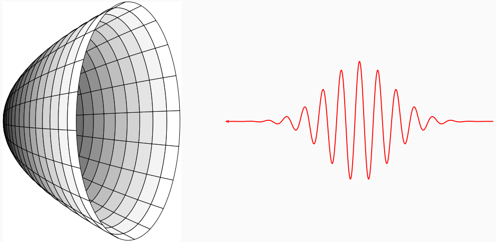

Parabolic mirror focusing a realistic laser beam.

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That's easy to model, no?

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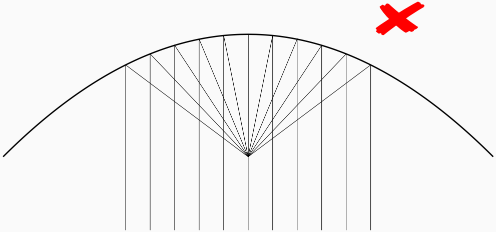


Ray focusing properties of a parabolic mirror.

P. Varga and P. Török, "Focusing of electromagnetic waves by paraboloid mirrors. I. Theory," J. Opt. Soc. Am. A 17, 2081–2089 (2000).

Stratton-Chu Integrals

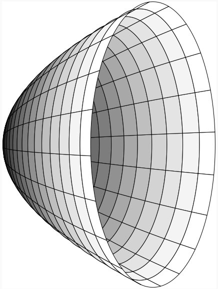
Stratton-Chu Equations

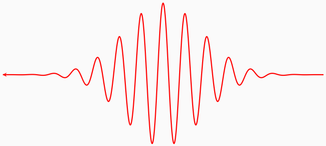
$$
\begin{aligned}
\boldsymbol{E}(\boldsymbol{r},k) &= \frac{1}{4\pi}\int_{\mathcal{S}} \{ik(\hat{\boldsymbol{n}} \times \boldsymbol{B}_{\mathcal{S}})G + (\hat{\boldsymbol{n}} \times \boldsymbol{E}_{\mathcal{S}}) \times \nabla_{\mathcal{S}} G + (\hat{\boldsymbol{n}} \cdot \boldsymbol{E}_{\mathcal{S}})\nabla_{\mathcal{S}} G\} \cdot d\mathcal{S} \\
&\quad + \frac{1}{4\pi i k}\oint_{\partial\mathcal{S}} (\nabla_{\mathcal{S}} G)\boldsymbol{B}_{\mathcal{S}} \cdot d\boldsymbol{\ell}, \\
\boldsymbol{B}(\boldsymbol{r},k) &= \frac{1}{4\pi}\int_{\mathcal{S}} \{-ik(\hat{\boldsymbol{n}} \times \boldsymbol{E}_{\mathcal{S}})G + (\hat{\boldsymbol{n}} \times \boldsymbol{B}_{\mathcal{S}}) \times \nabla_{\mathcal{S}} G + (\hat{\boldsymbol{n}} \cdot \boldsymbol{B}_{\mathcal{S}})\nabla_{\mathcal{S}} G\} \cdot d\mathcal{S} \\
&\quad - \frac{1}{4\pi i k}\oint_{\partial\mathcal{S}} (\nabla_{\mathcal{S}} G)\boldsymbol{E}_{\mathcal{S}} \cdot d\boldsymbol{\ell},
\end{aligned}
$$

F. Fillion-Gourdeau, C. Lefebvre, and S. MacLean, “Scheme for the detection of mixing processes in vacuum,” Phys. Rev. A 91, (2015).

$$\mathbf{E}(\mathbf{r}, k) = \int_{\mathcal{S}} \mathbf{f}_1(\mathbf{E}, \mathbf{B}, G) \cdot d\mathcal{S} + \oint_{\partial\mathcal{S}} \mathbf{f}_2(\mathbf{E}, \mathbf{B}, G) \cdot d\boldsymbol{\ell}$$

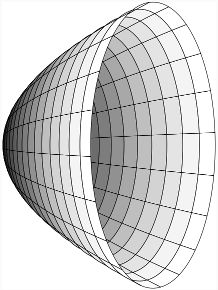
$$\mathbf{B}(\mathbf{r}, k) = \int_{\mathcal{S}} \mathbf{f}_3(\mathbf{B}, \mathbf{E}, G) \cdot d\mathcal{S} + \oint_{\partial\mathcal{S}} \mathbf{f}_4(\mathbf{B}, \mathbf{E}, G) \cdot d\boldsymbol{\ell}$$

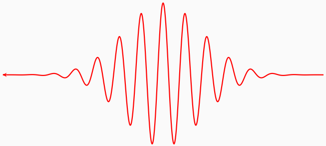




Problem Statement

Numerically evaluate the Stratton-Chu integrals for any mirror $\mathcal{S}$ and for any incident laser field $\boldsymbol{E}_{\mathcal{S}}$, $\boldsymbol{B}_{\mathcal{S}}$.





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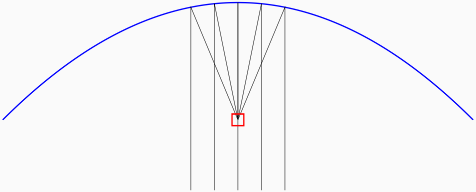
Advantages

- Multi-scale algorithm.



Meter-scale parabola, nanoscale focal spot.

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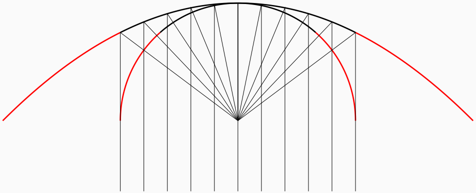

Ray optics approximation.

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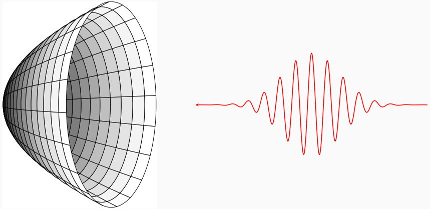

Reflection problem.

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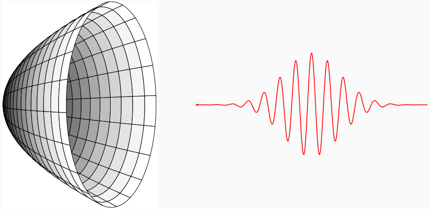

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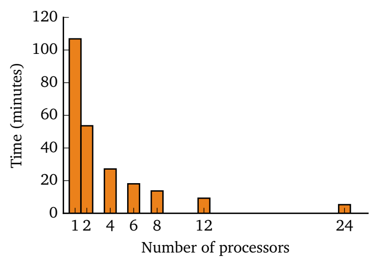

Run times for relatively small simulations.

Disadvantages

- Relatively heavy to evaluate numerically (relative to analytical models [$\sim$ ms]).

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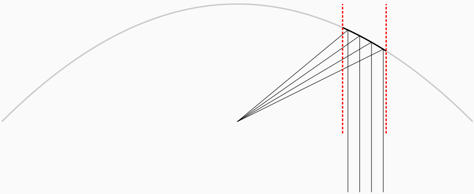

Offset parabola. Difficult to represent with current data structures.

Disadvantages

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- Difficulty in generalizing to arbitrary mirror geometries.

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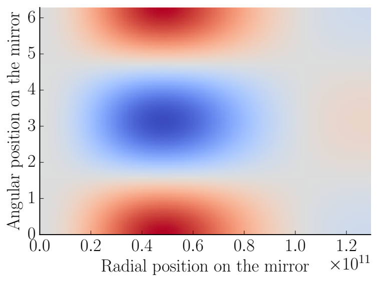

Sample of integrand on a parabolic mirror for an observation point at the geometrical focus.

Disadvantages

- Relatively heavy to evaluate numerically (relative to analytical models [$\sim$ ms]).
- Difficulty in generalizing to arbitrary mirror geometries.
- Rapidly oscillating integrals must be evaluated far from the focus.

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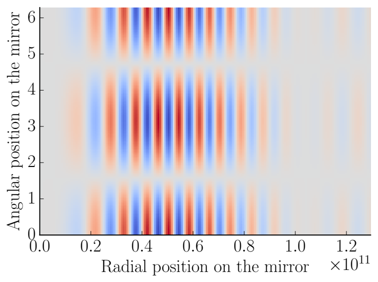

Sample of integrand on a parabolic mirror for an observation point 10 wavelengths away of the geometrical focus.

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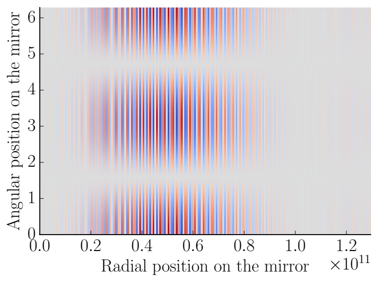

Sample of integrand on a parabolic mirror for an observation point 100 wavelengths away of the geometrical focus

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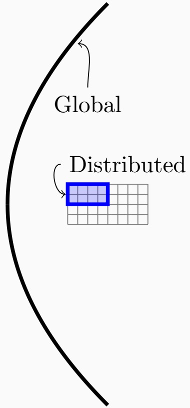
Computational Features

MPI Parallelization

- Focal spot decomposed on multiple processors.

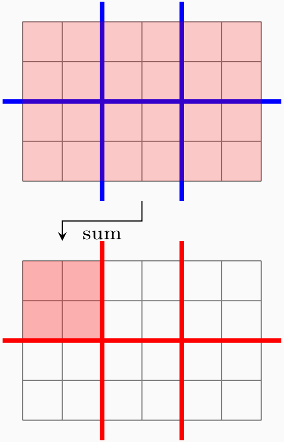


Parabolic mirror (integration domain) is global. Focal spot (result of the integral) is distributed.

Features of the program

MPI Parallelization

- Focal spot decomposed on multiple processors.
- Integration between two decomposed domains.

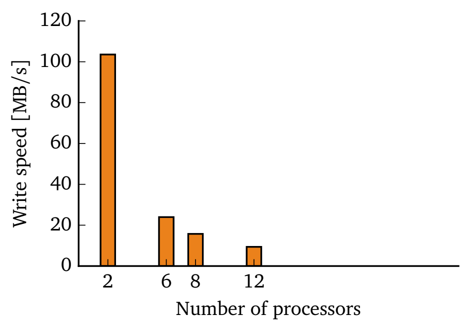


Data on one processor is the result of integration over all processors.

Features of the program

MPI Parallelization

- Focal spot decomposed on multiple processors.
- Integration between two decomposed domains.
- Fast parallel output with HDF5.



Estimated write speeds for a small simulation. Open|SpeedShop for exclusive time in `__write` divided by size of output.

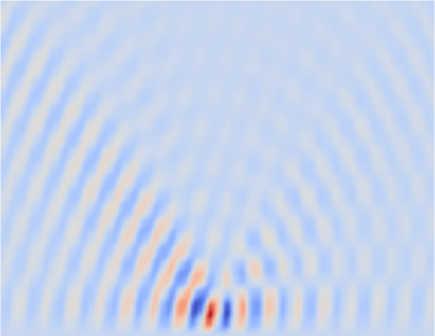
Features of the program

```xml
<?xml version="1.0" ?>
<!DOCTYPE Xdmf SYSTEM "Xdmf.dtd" []>
<Xdmf Version="2.1">
<Domain>
<Grid Name="Simulation" GridType="Collection"
      CollectionType="Spatial">
<Grid Name="EMFieldMany" GridType="Uniform">
<Topology TopologyType="3DRectMesh"
          Dimensions="40 40 40 "/>
<Attribute AttributeType="Scalar"
           Name="Er-0-amplitude" Center="Node">
<DataItem Dimensions="40 40 40 " NumberType="Float"
          Precision="8" Format="HDF5" Endian="Big">
  Field_reflected.hdf5:/field/Er-0/amplitude
</DataItem>
</Attribute>
<Geometry GeometryType="VXVYVZ">
<DataItem Name="r" Dimensions="40" NumberType="Float"
          Precision="8" Format="HDF5" Endian="Big">
  Field_reflected.hdf5:/coordinates/r
</DataItem>
...
</DataItem></Geometry></Grid></Grid></Domain>
</Xdmf>
```

Sample XDMF code.

Visualization

- Use of XDMF to visualize HDF5 data in ParaView.



Single frequency component at the geometrical focus.

Visualization

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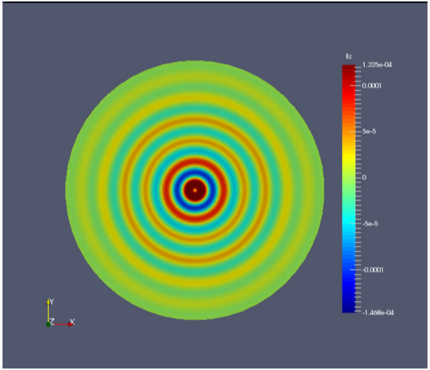
- Access to both the frequency and time domains.



Snapshop of the field in the geometrical focal spot.

Visualization

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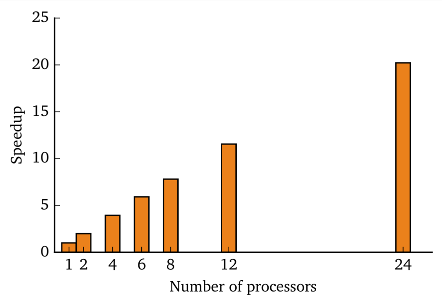
- Access to both the frequency and time domains.

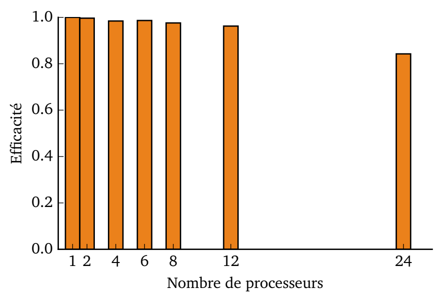
Results

Parallel Efficiency/Scaling

Low amount of communication needed makes for an ideal algorithm for parallelization. It has strong scaling.

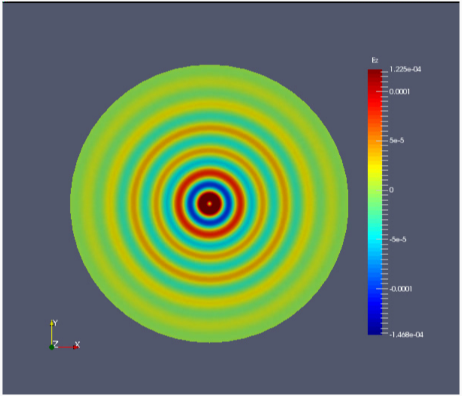


Speed up as a function of the number of processors for a given configuration.

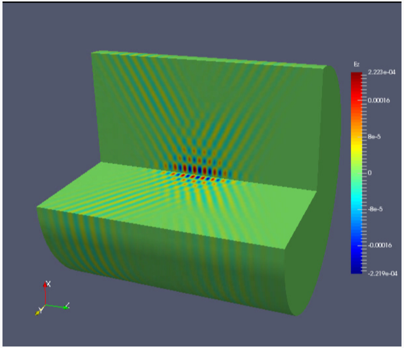


Parallel efficiency as a function of the number of processors for a given configuration.

Temporal evolution of the field



Snapshop of the field in the geometric focal plane.



3D snapshot of the field in the focal region.

Acknowledgements

Stellar Research Group

- Steve MacLean
- Jean-Claude Kieffer
- François Fillion-Gourdeau
- Catherine Lefebvre
- Denis Gagnon
- Stéphane Payeur
- Sylvain Fourmaux
- Amélie Lachapelle

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